FLAME FRONT

Combustion waves can propagate over a wide range of burning velocities that differ by more than three orders of magnitude for the same mixture. At one end of the velocity spectrum, we have a laminar flame (deflagration) that propagates at a typical velocity of about half a meter per second for common fuel–air mixtures at normal conditions. At the other end, the combustion wave propagates as a detonation whose speed is of the order of a couple of thousand of meters per second in the same mixture. In between these limits, we have an almost continuous range of turbulent burning velocities. Both laminar flames and detonation wave are intrinsically unstable and have the morphology of a transient cellular structure (Figure 1).

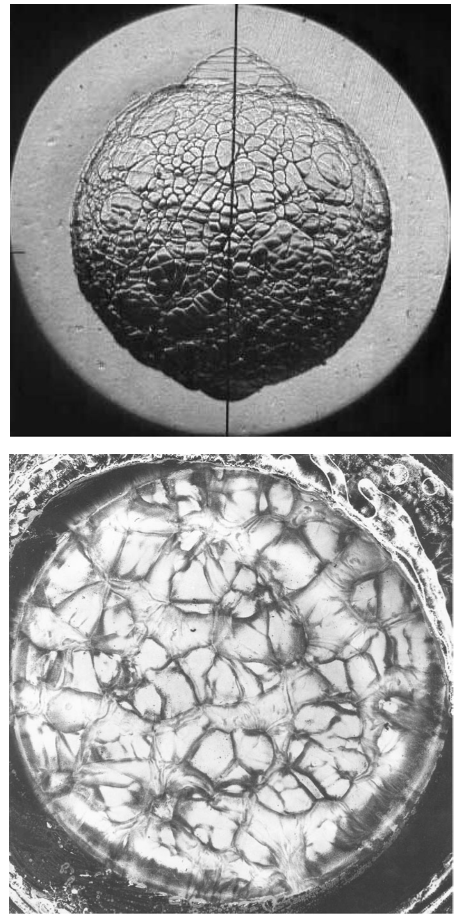

Figure 1. Cellular structures of laminar flame and detonation front. (a) Laminar outward propagating spherical flame (rich dimethyl ether–air at equivalence ratio of 1.2 and 10 atm) (Ju, 2003) (b) Detonation: unstable cellular detonation front AQ: 1
as recorder upon reflection from a soot coated glass plate (C_2H_2–O_2 mixture at 10 mmHg) (Lee, 2003). AQ: 2

A laminar flame is essentially an isobaric diffusion–reaction wave. Its propagation speed is determined by the rate of diffusion transport of heat from the reaction zone to the cold unburned mixture and the characteristic time of heat release of the chemical reactions. A laminar flame speed (S_L) is proportional to the square root of the product of thermal diffusivity (D_{th}) and the reaction rate (w_r) (i.e., $S_L \sim \sqrt{D_{th} w_r}$). The reaction rate is given by $w_r \sim \exp(-E/RT_f)$ in which E is the activation energy and T_f is the flame temperature. Since the activation energy is very large in general, the reaction rate is extremely temperature sensitive. Thus, any fluctuation in the flame temperature will result in a large variation in the reaction rate leading to the development of instability of the flame front. Due to the large density and temperature changes across the flame, a strong thermal expansion of the burned gas results from the conservation of mass. The flame as a strong density interface as well as an expansion wave is subject to a number of dynamic instability mechanisms in the presence of an acceleration field. Furthermore, competition between heat and mass diffusion across the flame results in thermal diffusion instability. Rapid density changes across the flame also give rise to acoustic wave generation that can couple with increase in burning rate to induce acoustic driven instability. Thus, in practice, there is a wealth of instability mechanisms that render laminar flames unstable. Various instability mechanisms can be at work simultaneously and can influence each other. However, historically, each instability mechanism was isolated and studied individually and was thus named after the original researchers.

The flame as a density interface is unstable when subjected to an acceleration field (for example, gravity). If the flame propagates upward, the light burned gas (ρ_b) is at the bottom and the heavy unburned gas (ρ_u) is on the top. The lighter fluid will be driven upward (i.e., buoyancy) whereas the heavier fluid is driven downward by gravity (g). This motion will destabilize the interface and hence small perturbations on the flame surface will grow with time (t). If the perturbed flame surface (F) is defined as $A\exp(\sigma t + \mathrm{i}kx)$, where k is the wave number and x the space coordinate, the disturbance growth rate σ can be determined from normal mode stability analysis as

$$\sigma = \sqrt{gk\gamma/(2-\gamma)}, \quad \gamma = 1 - \rho_b/\rho_u. \tag{1}$$

Therefore, instability occurs at all wavelengths but growth is faster at shorter wavelengths. This phenomenon was first discovered by Lord Rayleigh (1883) and later by Geoffrey Ingram Taylor (1950) and thus it is referred to as the Rayleigh–Taylor instability. This instability is common to all density gradient fields in the presence of an acceleration field normal to it.

Due to the density change across the flame, the flow velocity increases as the density ratio since the flame is approximately isobaric. The expansion across the front will induce a divergent flow field ahead of a curved flame (Williams, 1985). This divergence slows down the local flow speed ahead of the curved flame front and assuming the flame speed to be constant, this will result in a growth of the curvature of the flame. This instability was first discovered by G. Darrieus (1938) and Lev Landau (1944). The Darrieus–Landau instability was obtained by treating the flame as a surface of discontinuity moving at a constant speed. In the limit of small density change, stability analysis gives the growth rate as

$$\sigma = \gamma k S_{\mathrm{L}}/2. \tag{2}$$

Therefore, it was concluded that a flame front is unstable to perturbations at all wavelengths, with growth rate proportional to wave number (i.e., perturbations grow faster for small wavelengths). Unfortunately, this conclusion was contrary to later laboratory observations of small-scale stable flames.

The deficiency of this model was the result of neglecting the finite thicknesses of the flame front, which influences the flame speed when the flame is curved. To include the effect of flame thickness, George Markstein (1951) proposed a phenomenological model by adding a modification of flame speed due to curvature and showed that curvature decreases the flame speed and tends to stabilize the flame at short wavelengths inhibiting its growth. A more rigorous derivation of the dispersion equation including diffusion effect on the flame speed (in the limit of small density jump) was given later by Gregory Sivashinsky (1983) as

$$\sigma = \frac{\gamma k S_{\mathrm{L}}}{2} - \left[\frac{\beta}{2}(\mathrm{Le}-1)+1\right] \times D_{\mathrm{th}}k^2 - 4\frac{D_{\mathrm{th}}^3}{S_{\mathrm{L}}^2}k^4 + \sqrt{gk\gamma/(2-\gamma)}, \tag{3}$$

where $D_{\mathrm{th}}k^2$ in the second term of Equation (3) represents the thermal relaxation via transverse thermal diffusion that stabilizes the flame. Le is the Lewis number (i.e., the ratio of thermal diffusivity to mass diffusivity) and $(\mathrm{Le}-1)$ in the second term denotes the competition between heat loss via thermal diffusion and the enthalpy gain via mass diffusion. As a result, flame temperature will increase or decrease if Le is less or larger than unity. Equation (3) shows that if $\mathrm{Le}-1$ is less than $=2/\beta$ (β is the reduced activation energy), diffusion transport will destabilize the flame. This is the mechanism of the cellular instability. The third term in Equation (3) is the thermal relaxation to the modification of flame temperature caused by the heat and mass diffusion (Clavin, 1985). Therefore, for long wavelength disturbance, the hydrodynamic instability dominates (Figure 1a). At short wavelengths, diffusion relaxation stabilizes the flame. At moderate wavelengths, the competition between heat and mass transfer induces cellular instability at small Le. At large Le, flame temperature is very sensitive to the mass diffusion of the deficient reactant. Coupling between the diffusion and temperature sensitive reaction yields pulsating and spinning waves. This traveling wave is often seen in lean propane–air flames.

By further considering the effect of flame curvature on flow field, Equation (3) can be normalized as an evolution equation of flame front

$$F_t + \tfrac{1}{2}(\nabla F)^2 + \nabla^2 F + 4\nabla^4 F = 0 \tag{4}$$

This is the so-called Kuramoto–Sivashinsky equation which was independently developed by Yoshiki Kuramoto (1976) for the study of phase turbulence in the Belousov–Zhabotinsky reaction and by Gregory Sivashinsky (1983) for thermal diffusive instabilities of flame fronts. This equation has also been used to model directional solidification and weak fluid turbulence. However, the assumption of small density jump used in Equations (3) and (4) is not rigorous in practical flames. A unified model considering large density jump and Le was obtained by Class Andreas et al. (2003).

Heat release by combustion results in an increase in the specific volume of the product gases and thus generates acoustic waves (Chu, 1956). The acoustic waves play two roles in affecting combustion, (1) inducing pressure-heat release coupling via pressure-dependent reactions, and (2) increasing flame surface area via the baroclinic torque (Meshkov instability). If the changes of pressure and chemical heat release are in phase, the acoustic instability occurs (Rayleigh, 1877; Markstein, 1953; Clavin, 2002). Lord Rayleigh first used this criterion (Rayleigh criterion) to explain the singing flame and Rijke's tone (where heating the bottom of a tube causes it to produce sound). The acoustic instability causes the major problems of noise and vibration in combustors (Putnam & Dennis, 1953). On the other hand, volumetric heat loss reduces the flame speed and changes the resident time of the emitting gases. This coupling triggers the radiation induced instability for weak flames (Ju et al., 2000).

At the upper limit of propagation of combustion waves, the propagation mechanism is not due to diffusion. The flame instability mechanisms discussed above are too slow to be relevant in detonation wave instability. A detonation wave is a supersonic compression wave where mixture is ignited by the adiabatic compression of the leading shock. The classical structure of a detonation wave was formulated by Yakov Zeldovich, John von Neumann, and W. Döring (ZND) independently in the early 1940s and consists of a leading shock followed by the reaction zone after a short induction length (Zeldouich, 1940; von Neumann, 1942; Döring, 1943). Gas dynamic theory gives the detonation wave speed as proportional to the square root of the chemical heat release and does not involve any non-equilibrium rate processes. Again due to the high-temperature sensitivity of the reaction rates, small temperature fluctuations due to variation of the leading shock speed will result in large variations in the induction length and reaction rates, hence the coupling between the energy release zone and the leading shock. The instability yields a transient three-dimensional cellular detonation front (Lee, 1984). The unstable cellular detonation front consists of an ensemble of interacting transverse shock waves with the leading shock front. The cell boundaries (Figure 1b) are formed by the intersections of the transverse shocks. Shock interactions (Mach reflections) also give rise to the formation of shear layers which lead to turbulence generation due to Kelvin–Helmholtz instability. Chemical reactions in cellular detonations occur in disjointed piecemeal zones embedded within the complex of interacting shocks and shear layers.

The instability of the laminar ZND detonation structure was demonstrated theoretically by standard normal mode stability analysis using the one-dimensional Euler equation (e.g., Erpenbeck, 1964; Lee & Stewart, 1990). In one dimension, unstable detonations are referred to as pulsating detonations that goes from harmonic oscillations near the stability limit to highly nonlinear and eventually to chaotic oscillations with the increase of the activation energy. By examining the bifurcation diagram (Ng et al., 2004), it is interesting to find that the path to higher instability mode follows closely the Feigenbaum route (Feigenbaum, 1983) of a period-doubling cascade observed in many nonlinear systems. One-dimensional pulsating detonation as well as two- and three-dimensional cellular detonations have been reproduced qualitatively via numerical simulation using the reactive Euler equations (Bourlioux et al., 1991; Short & Stewart, 1999). However, the detailed description of the turbulent structure and chemical reactions requires resolutions far beyond current computing capabilities.

In between the two limits of laminar flames and detonations, there is a continuous range of flame speeds that depend on turbulence. The morphology of a turbulent flame is a time-dependent cellular or wrinkled surface. Turbulent flame is in fact an unstable flame and the effect of turbulence is to increase the burning rate via faster transport and increase in burning surface area. In the limit of very intense turbulence where mixing and reaction rates are comparable, auto-ignition may result and thus the mechanism becomes similar to that of a detonation. It differs only in the manner in which auto-ignition is achieved by turbulent mixing of fresh mixture with hot products or by adiabatic heating of the leading shock. Thus nature tends to maximize the burning rate of a mixture and instability is a route to optimize the burning rate for given initial and boundary conditions.

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See also **Candle; Explosions; Forest fires; Kuramoto–Sivashinsky equation; Reaction–diffusion systems; Zeldovich–Frank-Kamenetsky equation**

Further Reading

Bourlioux, A., Majda, A.J. & Roytburd, V. 1991. Theoretical and numerical structure for unstable one-dimensional detonations. *SIAM Journal of Applied Maths*, 51: 303–343

Class Andreas G., Matkowsky, B.J. & Klimenko, A.Y. 2003. Stability of planar flames as gasdynamic discontinuities. *Journal of Fluid Mechanics*, 491: 51–63

Clavin, P. 1985. Dynamic behaviour of premixed flame fronts in laminar and turbulent flows. *Progress in Energy and Combustion Science*, 11: 1–39

Clavin, P. 2002. Dynamics of combustion fronts in premixed gases: from flames to detonation. *Proceedings of the Combustion Institute*, 29: 569

Chu, B.T. 1956. Stability of systems containing a heat source—the Rayleigh criterion, National Advisory Committee for Aeronautics research memorandum, RM56D27

Darrieus, G. 1938. Propagation d'un front de flame, unpublished work presented at Paris: La Technique Moderne and le Congrès de Méchanique Appliquée

Döring, W. 1943. On detonation processes in gases. *Annals of Physics, Leipzig*, 43: 421–436

Erpenbeck, J. 1964. Stability of idealized one-reaction detonations. *Physics of Fluids* 7: 684–696

Feigenbaum, M. 1983. Universal behaviour in nonlinear systems. *Physica* D, 7: 16–39

Ju, Y., Law, Chung K., Maruta, K. & Niioka, T. 2000. Radiation induced instability of stretched premixed flames. *Proceedings of the Combustion Institute*, 28: 1891–1900

Kuramoto, Y. & Tsuzuki, T. 1976. Persistent propagation of concentration waves in dissipative media far from thermal equilibrium. *Progress of Theoretical Physics*, 55: 356–369

Landau, L.D. 1944. On the theory of slow combustion. *Acta Physiocochimica URSS* 19: 77–85

Lee, J.H.S. 1984. Dynamic parameters of gaseous detonation. *Annual Reviews of Fluid Mechanics*, 16: 311–316

Lee, H. & Stewart, D. 1990. Calculation of linear detonation instability: one-dimensional instability of plane detonation. *Journal of Fluid Mechanics*, 216: 103–132

Markstein, G.H. 1951. Experimental and theoretical studies of flame front instability. *Journal of the Aeronautical Sciences*, 18: 199–209

Markstein, G.H. 1953. Instability phenomena in combustion waves. *Proceedings of the Combustion Institute*, 14: 44–59

Ng, H.D., Higgins, A.J., Kiyanda, C.B., Radulescu, M.I., Lee, J.H.S., Bates, K.R. & Nikiforakis, N. 2004. Nonlinear dynamics and chaos analysis on one-dimensional pulsating detonations, Combustion Theory & Modeling, submitted AQ: 3

Ng, H.D., Radulescu, M.I., Higgins, A.J., Nikiforakis, N. & Lee, J.H.S. 2004. Numerical investigation of the instability for one-dimensional Champan-Jouguet detonations with chain-branching kinetics, Combustion Theory & Modeling, submitted

AQ: 4

Putnam, A.A. & Dennis, W.R. 1953. A study of burner oscillations of the organ-pipe type. *Transaction of the ASME*, 75: 15–28

Rayleigh, L. (John William Strutt). 1877. *The Theory of Sound*, vol.2, London: Macmillan; reprinted New York: Dover, 1945, p. 226

Rayleigh, L. (John William Strutt). 1883. Investigation of the character of the equilibrium of an incompressible heavy fluid of variable density. *Proceedings of the London Mathematical Society*, 14: 170–177

Short, M. & Stewart, D. 1999. Multi-dimensional stability of weak-heat-release detonation. *Journal of Fluid Mechanics*, 382: 103–135

Sivashinsky, G.I. 1983. Instabilities, pattern formation, and turbulence in flames. *Annual Reviews of Fluid Mechanics*, 15: 179–199

Taylor, G.I. 1950. The instability of liquid surfaces when accelerated in a direction perpendicular to their Planes. *Proceedings of the Royal Society of London* A, 201: 192–196

von Neumann, J. Theory of detonation waves, Proj. Report No. 238, OSRD report No.549, 1942; In Von Neumann, *Collected Works*, vol. 6, edited by A.J. Taub, Oxford: Pergamon Press, 1963

Williams, F.A. 1985. *Combustion Theory*, 2nd edition, New York: Benjamin Cumming

Zeldovich, Y.B. 1940. On the theory of the propagation of detonations in gaseous systems. *Experimental and Theoretical Physics SSSR*, 10: 542

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