An analysis of sub-limit flame dynamics using opposite propagating flames in mesoscale channels

Yiguang Ju*, C.W. Choi

From the Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544, USA

Received 10 October 2002; received in revised form 10 February 2003; accepted 10 February 2003

Abstract

The excess enthalpy flames and their dynamics below the flammability limit are studied by considering two flames that propagate in opposite directions in parallel channels. The model enables the coupling between the external heat loss, convection preheating, diffusion transport and finite rate chemistry. Analytical expressions for the flame temperature, separation distance, and extinction limit are obtained. The results show that flame extinction can be caused by the external heat loss without heat conduction of inner wall in the streamwise direction. The heat recirculation across the separating wall dramatically increases the flame speed and extends the flammability limit. It is shown that the maximum and minimum flame speeds corresponding respectively to the fast and slow flame modes exist at all separation distances between the two flames. It is found that the flame can adjust its separation distance to adapt to the variation of heat loss, heat recirculation and fuel concentration. There exists a maximum flame separation distance beyond which sub-limit flame does not exist. The results also showed that heat recirculation significantly extends the flammability limit. Furthermore, at low fuel concentrations, the flame can be stabilized in a narrow range of separation distance. The present study not only generalized the previous analyses of the heat recirculation flames but also provided a model for the study and control of sub-limit flames in micro power devices and reactors. © 2003 The Combustion Institute. All rights reserved.

Keywords: Microscale combustion; Heat recirculation; Excess enthalpy combustion

1. Introduction

With the recent development of micro power generators [1,2], micro sensors, and reactors, there is a renewed interest in the study of the dynamics and control of excess enthalpy flames. It is well known that with the decrease of the combustor scale, the increase of large surface/volume ratio dramatically increases the efficiency for catalytic reaction and provides chances to control the combustion process. However, the increase of surface/volume ratio also results in an increase of heat loss to the wall and leads to flame extinction, instability, and the reduction of thermal efficiency. Therefore, to minimize the heat loss and to extend the flammable region, the technology to generate excess enthalpy flames by recirculating the thermal energy in the exhaust gas for the preheating of the unburned mixture attracts great attentions.

The excess enthalpy flames were originally proposed by Weinberg [3], and Lloyd and Weinberg [4,5]. The swissroll type combustor was used to preheat the fresh unburned mixture with the burned gas. The experimental and analytical results showed that: 1) the flammability limit can be dramatically extended to low fuel concentration; and, 2) there is a minimum flow rate below which flame is quenched by heat loss and a maximum flow rate above which
flame is quenched by the limit of characteristic flow time for the heat transfer between the burned and unburned gases. Unfortunately, quantitative analyses on flame extinction and limit mechanism were not conducted. Based on Weinberg’s concept, Takeno and co-workers [6,7] proposed a so called direct method to generate excess enthalpy flames by inserting a porous rod into the flame zone to provide the heat conduction directly from burned zone to unburned zone. Their computational and numerical studies showed that it is possible to burn mixtures as equivalence ratio of 0.151 at large flow rates. Although this concept may have applications in ultra-combustion of porous media [8,9], the use of a porous rod dramatically increases the weight of the combustor and is less efficient in heat transfer compared to that of swissroll design. Another excess enthalpy flames with direct radiation reabsorption of CO₂ diluted mixture was studied by Ju, Masuya and Ronney [10].

The only modeling study of quantitative extinction limits in the swissroll type heat recirculation burners was conducted by Jones, et al. [11]. The analysis was based on the global energy balance between the heat release, heat exchange, and heat loss, and employed the minimum reaction temperature for flame extinction. Although this simple model provided reasonable explanations to the experimental results [1,2], the requirement of specifications of the minimum reactor temperature and the amount of heat loss greatly limits its quantitative and even qualitative prediction. In fact, except for these two parameters, flame extinction also depends on the diffusion transport, rate of heat transfer between the unburned and burned gases, and the finite rate chemistry. To improve this analysis, Ronney [12] proposed a U-shaped counter current heat recirculation combustion model (shown in Fig. 1a). The unburned gaseous mixture flows into a well stirred reactor and burns homogeneously there. The exhaust gas preheats the unburned mixture through convection on the both sides of the separating wall. This model dramatically simplified the complex swissroll excess enthalpy flames and considered the wall heat transfer and finite chemistry (in the WSR). The analysis [12] demonstrated the extinction limits at maximum and minimum flow rates when there is heat conduction in solid wall. Unfortunately, as described by the author, this model is not appropriate for flames away from the limits. If the flame has larger burning velocity than the incoming velocity, it will flash back. Otherwise, it will be blown out. Furthermore, the analysis also neglects the mass and thermal diffusion transports in flames, which dramatically affects the thickness of the preheating zone and consequently affects the in-

<table>
<thead>
<tr>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
</tr>
<tr>
<td>$H_0$</td>
</tr>
<tr>
<td>$h_{wi}$</td>
</tr>
<tr>
<td>$h_{wo}$</td>
</tr>
<tr>
<td>$K$</td>
</tr>
<tr>
<td>$\kappa$</td>
</tr>
<tr>
<td>$K_0$</td>
</tr>
<tr>
<td>$\kappa_c$</td>
</tr>
<tr>
<td>$\chi$</td>
</tr>
</tbody>
</table>

---

Fig. 1. a): the counter-current heat recirculation burner, b): the opposite propagating flames in parallel channels. WSR is the well stirred reactor. $H$ denotes the heat transfer rate through in inner wall, and $K$ is the heat transfer through the external walls. The shade region denotes the hot products.
ternal heat transfer between the burned and unburned gases and external heat loss to environment.

In micro combustion devices, stabilization of sub-limit flames and its position control becomes particularly important. Therefore, it is necessary to understand the flame stabilization and propagation properties in a wide range of equivalence ratio below the fundamental flammability limit (limit without heat recirculation). Unfortunately, the models in above cannot appropriately describe this phenomena.

The present study is motivated by the above discussions. We here present a model (Fig. 1b) to analyze the excess enthalpy flame dynamics below the fundamental flammability limit. The model involves two flames propagating in opposite directions (left and right) respectively in the lower and upper channels separated by a thin wall and having the same fuel concentrations and heat loss intensity. This model allows the consideration of the energy conservation between the chemical heat release, internal convection heat transfer, external heat loss, finite rate reaction and the propagation of flames in a wide range of equivalence ratio. The flame separation distance is defined as the distance of the two flames \((x_{f2}-x_{f1})\) as illustrated in Fig. 1. It is easy to understand that in the limit of zero flame separation distance, the present model reduces to Ronney’s model (Fig. 1a), but no assumption of WSR is needed. In the following section, the mathematical model and asymptotic analysis are described. This is followed by the analysis and the discussions of the results.

2. Mathematical models

We consider two flames propagating oppositely in parallel channels separated by a thin wall (Fig. 1b). The two flames are coupled together by heating each other with their burned gases via the convective heat transfer across the separating wall. The convection heat transfer rate is denoted by \(H\). In addition, both flames are subject to external heat losses, \(K\). It is understandable that, if the fuel concentration is above the fundamental flammability limit (the limit of a free propagating planar flame with the same heat loss), the two flames can propagate freely in the two channels. However, when the fuel concentration is below the fundamental flammability limit, the flames can only stand within a separation distance, \(x_{f2}-x_{f1}\). An increase of flame separation distance results in larger heat loss and causes the flames to move back toward their original position because of the reduction of flame speed. On the other hand, if the flame is perturbed to move closer, it will have smaller heat loss and larger flame speed to increase its separation distance resulting in a self-stabilized flame. Furthermore, in the limit of zero flame separation distance, \(x_{f2} = x_{f1}\), the flame model becomes similar to the counter current flow burner (Fig. 1a) as the flames approach their extinction limit. Therefore, the present model provides an ideal flame geometry to study the dynamics of the sub-limit excess enthalpy flames in a broad range of equivalence ratio.

The flame propagation in the current model is two-dimensional in nature. However, if we limit our interest in mesoscale to microscale burners with channel width and wall thickness much smaller compared with the channel length scale, we can simplify the problem to the one-dimensional problem \([11–13]\). We further assume that the flow in each channel is laminar and is fully developed, so its heat transfer can be represented by a constant Nusselt number between 7 and 9. In addition, we only consider the case in which the inner wall is very thin and its heat conductivity is very high so that the heat resistance due to convection heat transfer is dominant. Based on these assumptions, we can isolate the coupling between the flame and the wall heat conduction in the streamwise direction. As such, the intensity of the internal convection heat transfer becomes into a key parameter governing the coupling of the two flames. In the limit of an insulated wall, the model reduces to two independent flame propagations in channels which were earlier analyzed by various researchers \([13–17]\). The external heat loss to the environment via the outer walls is due to natural or forced convection. A constant heat convection heat transfer coefficient also applies here \([18]\).

By introducing the conventional constant properties and thin film assumptions, the governing equations for the steady-state energy and species conservations in both channels at a given flow speed can be written as below

\[
\begin{align*}
U \frac{d\theta_1}{dx} &= \frac{d^2\theta_1}{dx^2} - H_w(\theta_1 - \theta_w) - K \theta_1 + \exp(\beta(\theta_f - 1)/2) \delta(x - x_{f1}) \\
U \frac{dY_1}{dx} &= \frac{1}{Le} \frac{d^2Y_1}{dx^2} - \exp(\beta(\theta_f - 1)/2) \delta(x - x_{f1}) \\
-U \frac{d\theta_2}{dx} &= \frac{d^2\theta_2}{dx^2} - H_w(\theta_2 - \theta_w) - K \theta_2 + \exp(\beta(\theta_f - 1)/2) \delta(x - x_{f2}) \\
-U \frac{dY_2}{dx} &= \frac{1}{Le} \frac{d^2Y_2}{dx^2} - \exp(\beta(\theta_f - 1)/2) \delta(x - x_{f2})
\end{align*}
\]
where \( x \) is the streamwise coordinate normalized by \( \sqrt{\lambda/\rho C_p U_{ad}} \); \( U \) the flame speed normalized by the adiabatic flame speed \( U_{ad} \); \( \theta \) the normalized temperature, \((T - T_w)/(T_{ad} - T_w)\); \( Y \) the fuel concentration normalized by the initial fuel concentration \( Y_{f0}; \) \( H \) the normalized rate of heat transfer across the internal separating wall, \( h_w \lambda/d(\rho C_p U_{ad})^2 \); \( K \) the normalized rate of heat loss through outside walls, \( h_w \lambda/d(\rho C_p U_{ad})^3 \); \( \beta \) the Zeldovich number, \( E(T_{ad} - T_w)/(R_u)^\beta \), \( Le \) the Lewis number, and \( \delta \) the Delta function. The subscripts of “\( h \)”, “\( w \)” and “\( in \)” represent the corresponding values of adiabatic flames, unburned mixture at infinity and at flame fronts, respectively. In addition, in above normalizations, \( d \) is the channel width, \( \rho \) the density, \( C_p \) the specific heat at constant pressure, \( \lambda \) the heat conductivity of gas, \( E \) the activation energy and \( R_u \) the universal gas constant. \( h_w \) and \( h_{in} \) are respectively the coefficients of convective heat transfer of inner and outer walls.

Since the channel width is less than 5 mm and the flow velocity is less than \( 1 \text{m/s} \), the flow inside the channel is laminar (Reynolds number \( < 500 \)). Therefore, for the fully developed flow, the Nusselt number is [18]

\[
Nu = \frac{h_w d}{\lambda} = 8.23
\]

(2)

For the heat conduction along the inner wall, the steady state governing equation of wall temperature, \( \theta_w \), is:

\[
\alpha_w \frac{\partial^2 \theta_w}{\partial x^2} - 2H_w C (\theta_1 + \theta_2 - 2\theta_w) = 0
\]

(3)

where \( \alpha \) is the thermal diffusivity and \( C \) is the product of thermal inertial ratio and the thickness ratio of gas layer to inner wall layer, \((\rho C_p/\rho_w C_w)(d/d_w)\). Since our interest is the case of thin inner wall \((d/d_w \ll 1)\) for sub-limit flames \((U_{ad} < 0.03 \text{ m/s})\), this leads to \(2H_w C \gg \alpha_w/\alpha \) and \( \theta_1 + \theta_2 - 2\theta_w \approx 0 \). This result is equivalent to the thermally thin wall assumption made by Ronney [12]. We employ this approximation in the present analysis.

The boundary conditions for infinite long channels are given below:

\[
x \rightarrow -\infty, \quad \theta_1 = \theta_2 = 0, \quad Y_1 = 1, \quad Y_2 = 0
\]

\[
x \rightarrow +\infty, \quad \theta_1 = \theta_2 = 0, \quad Y_1 = 0, \quad Y_2 = 1
\]

(4)

3. Analyses

Let us choose the origin of the coordinate at the flame location of channel 1 \((x_{fl} = 0)\). The flame in channel 2 is located at \(x_{fl} = L\). Here, \( L \) can be either positive or negative. The positive sign implies that the flame in channel 2 is on the right hand side of the flame in channel 1. Using the boundary condition, the frozen solution of the fuel concentrations in channels 1 and 2 can be obtained as:

\[
Y_1 = 1 - \exp(-LeUx), \quad x < 0; \quad Y_1 = 0, \quad x > 0
\]

(5)

and

\[
Y_2 = 1 - \exp(-LeU(x - L)), \quad x > L; \quad Y_2 = 0, \quad x < L
\]

(6)

By submitting \( \theta_1 + \theta_2 - 2\theta_w = 0 \) into Eq. 1 and defining \( H = H_w/2 \), the governing equations for the coupling between \( \theta_1 \) and \( \theta_2 \) can be written as:

\[
(D^2 - UD - H - K)\theta_1 + H\theta_2 = 0
\]

\[
(D^2 + UD - H - K)\theta_2 + H\theta_1 = 0
\]

(7)

where \( D = d/dx \). The general solutions for above equations can be obtained as:

\[
\theta_1 = c_1 \exp(M_1 x) + c_2 \exp(M_2 x)
\]

\[
+ c_3 \exp(-M_1 x) + c_4 \exp(-M_2 x)
\]

\[
\theta_2 = c_1' \exp(M_1 x) + c_2' \exp(M_2 x)
\]

\[
+ c_3' \exp(-M_1 x) + c_4' \exp(-M_2 x)
\]

(8)

where:

\[
M_1 = \sqrt{\frac{2(H + K) + U^2 + \sqrt{(U^2 + 2(H + K))^2 - 8HK - 4K^2}}{2}}
\]

(9-1)

\[
M_2 = \sqrt{\frac{2(H + K) + U^2 + \sqrt{(U^2 + 2(H + K))^2 - 8HK - 4K^2}}{2}}
\]

(9-2)

\[
c_1' = \frac{UM_1 - M_1^2 + H + K}{H} c_1
\]

\[
c_2' = \frac{UM_2 - M_2^2 + H + K}{H} c_2
\]

(9-3)

\[
c_3' = \frac{-UM_1 - M_1^2 + H + K}{H} c_3
\]

\[
c_4' = \frac{-UM_2 - M_2^2 + H + K}{H} c_4
\]

(9-4)

By integrating Eq. 1 across the flame fronts, we can
obtain the following jump conditions at \( x_f \) and \( x_f^* \) [19]

\[
\left[ \frac{d\theta_1}{dx} + \frac{1}{Le} \frac{dY_1}{dx} \right]_+ = 0, \quad \theta_1^+ = \theta_1^- = -Le \exp[\beta(\theta_f - 1)/2] \quad \text{at } x = x_f
\]

and

\[
\left[ \frac{d\theta_2}{dx} + \frac{1}{Le} \frac{dY_2}{dx} \right]_+ = 0, \quad \theta_2^+ = \theta_2^- = -Le \exp[\beta(\theta_f - 1)/2] \quad \text{at } x = x_f^*
\]

3.1. Solution in the limit of flame separation

In this case, the solution of channels 1 and 2 can be given, respectively, for \( x < 0 \):

\[
\theta_1 = c_1 \exp(M_1x) + c_2 \exp(-M_2x),
\]

\[
\theta_2 = \frac{UM_1 - M_1^2 + H + K}{H} c_1 \exp(M_1x)
+ \frac{UM_2 - M_2^2 + H + K}{H} c_2 \exp(M_2x)
\]

(12)

\[
c_1 + c_2 = \frac{UM_1 - M_1^2 + H + K}{H} c_1 + \frac{UM_2 - M_2^2 + H + K}{H} c_2
\]

\[
c_1M_1 + c_2M_2 - U = -M_1 \frac{UM_1 - M_1^2 + H + K}{H} + M_2 \frac{UM_2 - M_2^2 + H + K}{H}
\]

(14)

Therefore, the flame temperature can be given as the function of flame speed

\[
\theta_f = c_1 + c_2 = \frac{\Omega_2 - \Omega_1}{\Omega_2} U
\]

\[
= \frac{\Omega_2 - \Omega_1}{2M_1 + \Omega_1(M_1 - M_2) - \Omega_1 \Omega_2} U
\]

(15)

where

\[
\Omega_i = \frac{UM_i - M_i^2 + K}{H}
\]

(16)

By further combining the jump condition and the solution of \( Y_1 \), we have the relation between the gradient of fuel concentration and the flame temperature

\[
U = \exp[\beta(\theta_f - 1)/2]
\]

(17)

and for \( x > 0 \):

\[
\theta_2 = c_1 \exp(-M_1x) + c_2 \exp(-M_2x)
\]

\[
\theta_1 = \frac{UM_1 - M_1^2 + H + K}{H} c_1 \exp(-M_1x)
+ \frac{UM_2 - M_2^2 + H + K}{H} c_2 \exp(-M_2x)
\]

(13)

By further using the jump conditions, the coefficients of \( c_1 \) and \( c_2 \) in above equations can be determined by

As such, Eqs. 15 and 17 determine the flame speed \( U \) and flame temperature \( \theta_f \).

In the limit of \( H = 0 \) (no heat recirculation) and small external heat loss \( K \), Eq. 16 yields:

\[
\Omega_1 = -H, \quad \Omega_2 \to \infty
\]

(18)

Thus, the flame temperature \( \theta_f \) becomes

\[
\theta_f = \frac{U}{2U\sqrt{1 + 4K/U^2} \approx 1 - \frac{2K}{U^2}}
\]

(19)

Substituting Eq. 19 into Eq. 17, we have

\[
U^2 \ln U^2 = -2\beta K
\]

(20)

Therefore, in the limit of zero heat recirculation, the present results reduce to the classical solution of Zeldovich and Spalding [14,15] of the free propagating planar flame subject to small heat loss.
3.2. Solution for flame separation distance \( L > 0 \)

In this case, the solution of channel 1 for \( x < 0 \) can be given as:

\[
\theta_1 = c_1 \exp(M_1x) + c_2 \exp(M_2x),
\]

where

\[
\begin{align*}
\theta_1 &= c_1^0 \exp(M_1x) + c_2^0 \exp(M_2x) + \frac{UM_1 - M_1^2 + H + K}{H} c_1 \exp(-M_1(x - L)) \\
&+ \frac{UM_2 - M_2^2 + H + K}{H} c_2 \exp[-M_2(x - L)]
\end{align*}
\]

and the solution for \( 0 < x < L \) becomes

\[
\theta_2 = c_1 \exp(M_1x) + c_2 \exp(M_2x),
\]

where

\[
\begin{align*}
\theta_2 &= c_1^0 \exp(M_1x) + c_2^0 \exp(M_2x) + \frac{UM_1 - M_1^2 + H + K}{H} c_1^0 \exp[-M_1(x - L)] \\
&+ \frac{UM_2 - M_2^2 + H + K}{H} c_2^0 \exp[-M_2(x - L)]
\end{align*}
\]

In addition, the solution for \( x > L \) can be similarly derived as

\[
\begin{align*}
\theta_1 &= \frac{UM_1 - M_1^2 + H + K}{H} c_1 \exp[-M_1(x - L)] + \\
&+ \frac{UM_2 - M_2^2 + H + K}{H} c_2 \exp[-M_2(x - L)]
\end{align*}
\]

By using the jump condition of flame temperature in channel 1 at \( x_f = 0 \)

\[
c_1 + c_2 = c_1^0 + c_2^0 + (1 + \Omega_1)\exp(M_1L)c_1^0 + (1 + \Omega_2)\exp(M_2L)c_2^0
\]

and flame temperature in channel 2 at \( x_s = L \),

\[
\begin{pmatrix}
1 + \Omega_1 & 1 + \Omega_2 & -1 - (1 + \Omega_1)\exp(M_1L) & -1 - (1 + \Omega_2)\exp(M_2L) \\
\Omega_1 & \Omega_2 & -(1 - \exp(M_1L))\Omega_1 & -(1 - \exp(M_2L))\Omega_2 \\
M_1 & M_2 & -M_1(1 - (1 + \Omega_1)\exp(M_1L)) & -M_2(1 - (1 + \Omega_2)\exp(M_2L)) \\
M_1\Omega_1 & M_2\Omega_2 & -(1 + \exp(M_1L))M_1\Omega_1 & -(1 + \exp(M_2L))M_2\Omega_2
\end{pmatrix} \begin{pmatrix}
c_1 \\
c_2 \\
c_1^0 \\
c_2^0
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
U \\
-U
\end{pmatrix}
\]

Therefore, the flame temperature and the flame speed are respectively given by:

\[
\theta_f = c_1 + c_2, \quad U = \exp[\beta(\theta_f - 1)/2]
\]

Note that, in the limit of \( L = 0 \), Eqs. 26 and 27 reduce to Eq. 20.

3.3. Solution for flame separation distance \( L < 0 \)

The solution of \( L < 0 \) is similar as that of \( L > 0 \) and the final results for flame temperature and flame speed are:

\[
\begin{pmatrix}
-1 + \Omega_1 & -1 + \Omega_2 & 0 & 0 \\
-\Omega_1 & -\Omega_2 & 0 & 0 \\
1 + \Omega_1 & 1 + \Omega_2 & -1 - (1 + \Omega_1)\exp(M_1L) & -1 - (1 + \Omega_2)\exp(M_2L) \\
\Omega_1 & \Omega_2 & -(1 - \exp(M_1L))\Omega_1 & -(1 - \exp(M_2L))\Omega_2 \\
-M_1(1 + \Omega_1) & -M_2(1 + \Omega_2) & -M_1(1 - (1 + \Omega_1)\exp(M_1L)) & -M_2(1 - (1 + \Omega_2)\exp(M_2L)) \\
-M_1\Omega_1 & -M_2\Omega_2 & -(1 + \exp(M_1L))M_1\Omega_1 & -(1 + \exp(M_2L))M_2\Omega_2
\end{pmatrix} \begin{pmatrix}
c_1 \\
c_2 \\
c_1^0 \\
c_2^0
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
-U \\
U
\end{pmatrix}
\]
and
\[ \theta_f = (1 + \Omega_1)c_1 + (1 + \Omega_2)c_2, \]
\[ U = \exp[\beta(\theta_f - 1)/2] \quad (29) \]

4. Results and discussions

In the following calculations, we choose the parameters to mimic the CH4-air combustion. The mixture density at 300K is \( \rho = 1.1774 \) kg/m\(^3\); the specific heat at constant pressure is \( C_p = 1005.7 \) J/kgK and the heat conductivity of gas phase is \( \lambda = 0.0675 \) W/mK. Although the present analysis includes non-uniformity Lewis number, since our focus here is not the Lewis number, we employ unity Lewis number in our prediction. The chemical heat release per unit mass of fuel is \( \dot{Q} = 10^{7} \) J/kg and the activation energy of chemical reaction is \( E = 1.2465 \times 10^5 \) J/molK. The reaction frequency factor, \( B \), is so chosen that it yields 41cm/s burning velocity at stoichiometric condition:
\[ U_{ad} = \frac{B}{\rho} e^{-E/2R_0T_0} \quad \text{and} \quad B = 10^k \text{kg/m}^2\text{s} \quad (30) \]

The channel width is 0.001m and the thin inner wall has a width of one tenth of the channel width. The resulting normalized convection heat transfer rate across the inner wall for fully developed flow is
\[ H_0 = 0.115 U_{ad}^{-2} \quad (31) \]
and the outside normalized heat loss rate at convection heat transfer rate of \( h_{wa} = 2 \) W/m\(^2\)s is
\[ K_0 = 8.3 \times 10^{-4} U_{ad}^{-2} \quad (32) \]
To model the effects of the preheating and heat loss on the flame propagation, the inner wall heat transfer and the outer wall heat loss are respectively varied with a factor of \( \chi \) and \( \kappa \)
\[ H = \chi H_0, \quad K = \kappa K_0 \quad (33) \]
Without specific notification, the fuel concentration is \( Y_f = 0.0365 \), which corresponds to fundamental flammability limit with the characteristic heat loss at \( \kappa = \kappa_c \).

4.1. Flame speed and extinction limit for zero flame separation distance

Fig. 2 shows the dependence of normalized flame speed as a function of the strength of external heat loss for various preheating rates. It is seen that for zero heat recirculation (\( \chi = 0 \)), flame speed decreases with the increase of heat loss. At a critical heat loss of \( \kappa = \kappa_c \), flame extinguishes and no flame exists for larger heat loss. For a given heat loss of \( \kappa_c \), this limit defines the flammability limit of the mixture. At the limit, the normalized flame speed is \( e^{-1/2} \).

When there is heat recirculation via the coupling of the two opposite propagating flames, it is seen that for \( \chi = 0.0001 \), the extinction limit is extended to larger heat loss. For the same heat loss, the heat recirculation increases the flame speed. Moreover, at small heat loss, the increase of flame speed becomes more significant. In addition, the increase of heat recirculation results in a dramatic extension of extinction limit and a rapid increase of flame speed. The conclusion of the increase of flame speed with heat recirculation agrees with the result of Ronney [11]. However, as shown in Fig. 2, flames can be quenched by external heat loss without the heat conduction loss of the inner wall. This result is different with the conclusion [12], in which it was concluded that flame extinction is only possible when heat conduction in the inner wall is considered. The difference in this study is that we did not use the WSR (well stirred reactor) assumption and included that effect of thermal diffusion on the flame temperature. As such, any external heat loss leads to flame extinction at limit conditions.

The comparison of structures of the preheating zones for flames in the upper and lower channels with and without heat recirculation at \( L = 0 \) is shown in Fig. 3. It is seen that without heat recirculation, flame structure (the convection-diffusion zone) is only determined by the thermal diffusion thickness. However, when there is heat recirculation, there exists a very long convective preheating zone in front of the convection diffusion zone. The mixture is preheated
in the convection preheating zone and yields a higher flame temperature. This result indicates that it is still reasonable to employ thin flame assumption, because the flame thickness is much less than the convection diffusion zone and also because the preheating increased the flame speed.

The dependences of flame temperature, flame speed and the heat loss on the preheating strength at the extinction limit are shown in Fig. 4. It is seen that although the flame temperature does not increase very much, the flame speed and the heat loss required for quenching the flame increase rapidly with the increase of preheating strength. At $\chi = 1.0$, it is seen that heat loss for flame extinction is 38 times higher than the natural convection, and the flame is quenched at a speed 6 times higher than that of adiabatic flame speed.

4.2. Flame speed and extinction limit for non-zero flame separation distance

The effect of flame separation distance on the flame speed is shown in Fig. 5. It is seen that for fuel concentration below the fundamental limit, there are two flame speeds for the same heat recirculation and heat loss. Furthermore, both the maximum flame speed and the minimum flame speed occur at zero flame separation ($L = 0$). The maximum flame speed occurs because the external heat loss from burned gas is minimized when the burned gas region between the two flames is zero. The minimum flame speed exists at the same condition because the heat loss between the flames at zero separation distance is minimized and allows burning with increased external heat loss at lower flame speed. The maximum flame speed and the minimum flame speed of the present analysis correspond respectively to the maximum and minimum flow rates reported in the previous experiments and analyses of the excess enthalpy flames [4–5,11–12]. Therefore, the present model can successfully predict the combustion limits of the excess enthalpy flames.

With the increase of flame separation distance, the burned gas heat loss between the flames increases and the preheating effect of burned gas to the unburned mixture decreases. As a result, the flame speed on the upper branch (fast mode) decreases while the flame speed on the lower branch (slow mode) increases. The mechanism of the decrease of the flame speed of fast mode and the increase of the flame speed of slow mode with the separation distance is similar to that of the dependence of flame speed on heat loss of the free propagating planar flame [14–17]. Moreover, it is seen that there is a
maximum flame separation distance at which the fast mode and slow mode merge together and beyond which flame does not exist. Therefore, there exists a maximum flame separation distance for the given fuel concentration and heat recirculation intensity.

On the other hand, the flame separation distance can be negative. The negative sign implies that the flame in upper channel is on the left side of the flame in lower channel. Fig. 5 shows that similar to the positive flame separation, there are also two flame modes at negative flame separation. As the flames move away from each other, the speed of the fast flame decreases while the speed of the slow mode increases. There exists also a maximum flame separation limit for the negative flame separation. Therefore, for a sub-limit mixture, the opposite propagating flames can only be held within a maximum flame separation distance. There are two flame speeds at all possible flame separation distance.

With the increase of heat recirculation intensity, the flame speed of fast mode increases and flame speed of slow mode decreases. In addition, the maximum flame separation distance increases. As such, with the change of the strength of heat recirculation, external heat loss and fuel concentration, the opposite propagating flames can adjust their separation distance and stabilize themselves. In other words, the opposite propagating flames are self-stabilized. This flame property provides a good opportunity for flame control at sub-limit conditions.

The maximum flame separation distance and their corresponding flame temperature as a function of heat recirculation for fuel concentration of 0.0365 are shown in Fig. 6. It is seen that the maximum flame separation distance increases rapidly at small heat recirculation. With further increase of heat recirculation, the maximum separation distance increases linearly. The flame temperature at extinction limit is below the adiabatic flame temperature.

The temperature distributions of upper and lower channels at small and large flame separation distances with difference fuel concentration (constant heat loss) are shown in Figs. 7 and 8. Fig. 7 shows that when the flame separation distance is small, there is a strong coupling between the two flames. It is seen that the lower flame (solid line) is preheated at a distance even larger than 10 times of the flame thickness ($\alpha U_{ad}$) although the actually flame speed is much larger than the adiabatic flame speed. In addition, there is an extensive heat transfer between the exhausted gas and the unburned fresh gas in the region between the two flame fronts. It is seen that there is a rapid drop of the burned gas temperature.
after the flame front due to this preheating effect. This explains why the flame speed at the fast mode is higher at small flame separation distance. Fig. 8 shows different temperature distributions in the limit of large flame separation. It is seen that there is no coupling of the burned gas and unburned mixture between the two flames. This is because the flame separation distance is so large that the burned gas is completely cooled down by the external heat loss before it can heat the other flame’s unburned mixture. The only role of the burned gas here is to heat the other flame’s burned gas and leads to increased heat loss. Therefore, the control of flame separation distance is important to improve the preheating effect.

4.3. Diagram of flammable region

The limit of the maximum flame separation distance as a function of equivalence ratio is shown in Fig. 9. Since the fundamental lean limit is $Y_f = 0.0365$, when the fuel concentration is above this fundamental limit, the two opposite propagating flames can freely propagate in the parallel channel. Therefore, the flame separation distance is infinite and the flame position control becomes difficult. However, when the fuel concentration is below the fundamental limit, flame separation distance becomes finite and convergent. At small heat recirculation ($\chi = 0.0006$), the maximum flame separation distance decrease dramatically with the decrease of equivalence ratio and the lean flammability limit occurs at zero flame separation distance. With the increase of the strength of heat recirculation, the lean flammability limit dramatically widens and the flame separation distance decreases slower with the decrease of equivalence ratio. In particular, for $\chi = 0.015$, it is seen that in a wide range of equivalence ratios (0.0225–0.0275), the maximum flame separation distance holds in a narrow range. This result indicates that flame position can be more easily controlled within a narrow range at very lean combustion. This flame property is very important for micro power generator and micro chemical reactor in which flame position needs to be accurately controlled.

Recently, we have experimentally confirmed that a stable flame can be obtained using this flame geometry. In addition, our experimental and analytical results showed that only one flame branch (positive flame separation distance is stable). Detailed reports of the stability research will be published in our forthcoming paper.

5. Conclusions

An opposite propagating flame geometry was presented to analyze the sub-limit excess enthalpy flame with heat recirculation across the inner wall. An explicit solution for the flame speed, temperature and separation distance between the two flames was obtained. The analysis showed that the present model not only can model the flame near the extinction limit but also extends the study of the flame dynamics for all equivalence ratios below the fundamental limit.

The results showed that flame can be quenched by the external heat loss without the heat conduction across the inner wall. The heat recirculation via the coupling of the flames yields a long convection heat transfer zone in front of the convection-diffusion zone of the flames, and dramatically increases the flame speed and extends the limit of the maximum heat loss. It is found that flames can adjust their separation distance to adapt to the variation of heat loss, heat recirculation and fuel concentration. It is also shown that there are two flame speeds for all the separation distances. Both the maximum flame speed of fast mode and the minimum flame speed of the slow mode occur at zero flame separation distance. The increase of flame separation distance respectively causes the decrease and increase of the flame speeds of the fast and slow modes. It is demonstrated that there is a maximum flame separation distance beyond which sub-limit flame does not exist.

The results also showed that the heat recirculation significantly extends the flammability limit. At low strength of heat recirculation, the flame separation distance is very sensitive to the fuel concentration. However, at large strength of heat recirculation, flame separation distance becomes increasingly less sensitive to the change of fuel concentration. In particular, at low fuel concentration, flame can be stabilized in a narrow range of separation distances. The
results suggest the possibility for flame position control using the current flame model. A future study will analyze on the stability of the flames.

Acknowledgments

Y. Ju thanks Prof. Chung K. Law at Princeton University and Prof. Takashi Niioka at Tohoku University for their support to publish this study. In addition, he thanks Prof. Paul D. Ronney at University of Southern California for helpful discussions.

References