Short communication

Combustion regimes subsequent to the reflection of a detonation from a perforated plate

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Abstract

A simple phenomenological approach is used to elucidate different combustion processes resulted from the interaction of a detonation with a perforated plate. The mathematical model is given by a simple nonlinear logistic difference equation, with parameters associated with the effect of turbulence, quenching and gasdynamics. Different nonlinear solutions of the logistic difference model provide an analog to the possible combustion regimes and they agree qualitatively with the experimental observations.
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1. Introduction

A detonation is a violent combustion-driven compression wave propagating at a supersonic velocity [1]. Its formation and subsequent propagation are governed by the nonlinear coupling between the chemical reaction and gasdynamic processes. In industrial settings, a detonation may accidentally form in a pipeline, and in order to prevent the propagation of the detonation further downstream, a detonation arrestor is used. An arrestor element is essentially an obstruction that is placed in the path of a detonation to quench the chemical reactions and to prevent the transmission of the detonation downstream. However, when an obstruction is placed in the path of a detonation, a variety of nonlinear combustion phenomena can be generated depending on the amount of blockage that the obstruction provides to the flow. In this light, it is worthwhile to simulate and elucidate the different flow fields that are generated by the reflection of a detonation with a perforated plate as a model problem.

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2. Experimental observation

A series of laboratory experiments on the interaction of a detonation with a perforated plate have been carried out in the past [2,3]. These experiments were conducted in a 4.5 m long and 150 mm in diameter detonation tube. Perforated plates with different configurations (e.g., hole diameter, hole spacing to hole diameter ratio, etc.) were used.

To summarize the experimental results, different possible flow fields observed upon the collision of a detonation with a perforated plate are illustrated in Fig. 1 showing schematically different wave trajectories and typical experimental Streak-Schlieren photographs taken downstream of the plate. Upstream of the perforated plate, a shock is reflected subsequent to the collision. Downstream, if ignition does not occur, only a transmitted shock is formed and is followed by a contact surface. This contact surface separates the combustion products that are discharged through the plate from the shocked gas behind the transmitted shock (Fig. 1(a)). The gasdynamics corresponds to a frozen regime and the strength of the transmitted shock will depend on the amount of blockage that is provided by the perforated plate. If the temperature of the combustion products discharged through the plate is sufficiently high, ignition can occur downstream at the interface that separates the combustion products from the unreacted mixture. A flame can begin to propagate away from the interface as shown schematically in Fig. 1(b). Under appropriate conditions, it is also observed that the combustion wave can propagate as a quasi-steady supersonic deflagration supported by the coupling between turbulence and chemical reactions or abruptly transit to a detonation [2,3] (Fig. 1(c) and (d)). It can be seen that there is a variety of possible combustion regimes resulting from the collision of a detonation against a perforated plate. At the present time, models that can predict these end results are not yet available.

3. A simple phenomenological model

Due to the complexity of the phenomena, investigation based on the theoretical analysis from first principle or “brute-force” approaches using multi-dimensional direct numerical simulation are very challenging. Hence,
the objective of this study is to consider a simple mathematical model to obtain better insight into the problem. We follow a phenomenological approach originally described by Moen et al. [4], and later by Chan et al. [5] to model turbulent flame acceleration in obstacle-filled tube. The mathematical model results in a logistic difference equation where each term can be associated with various physical effects such as turbulence, quenching and gasdynamics. As pointed out by Lord May [6], equation of logistic type includes many non-linear dynamics and can often capture the essence of a whole class of real world phenomena, e.g., population dynamics. While having simple form, the resulting logistic model should give some understanding for the present reactive flow problem

For an infinite decimal time \( \Delta t \), the local flame velocity at time \( \Delta t^n \) is assumed to be function of the turbulent intensity at \( \Delta t^{n-1} \), i.e.,

\[
\tilde{S}_f^n = f(\tilde{u}^{n-1}).
\]

Expanding the above function in a Maclaurin series, we get:

\[
\tilde{S}_f^n = f(\tilde{u}^{n-1}) = f(0) + f'(0)\tilde{u}^{n-1} + \frac{f''(0)}{2!}\tilde{u}^{n-1^2} + \frac{f'''(0)}{3!}\tilde{u}^{n-1^3} + \ldots
\]  

(2)

We further assume that the turbulent fluctuation depends on the local flame velocity at the same time.

\[
\tilde{S}_f^n = f(\tilde{u}^{n-1}) = g(\tilde{S}_f^{n-1}).
\]

(3)

The series expansion can then be written as:

\[
\tilde{S}_f^n = g(\tilde{S}_f^{n-1}) = g(0) + g'(0)\tilde{S}_f^{n-1} + \frac{g''(0)}{2!}\tilde{S}_f^{n-1^2} + \frac{g'''(0)}{3!}\tilde{S}_f^{n-1^3} + \ldots
\]

(4)

that has a general form of a logistic equation. Neglecting higher order term and using some phenomenological arguments as described by Chan et al. [5] based on the feedback mechanism between various competing effects, different coefficients are replaced as follows:

\[
\tilde{S}_f^n = \tilde{S}_{f0} + \gamma\tilde{S}_f^{n-1} - \omega\tilde{S}_f^{n-1^2} + \delta\tilde{S}_f^{n-1^3}.
\]

(5)

The constant term \( \tilde{S}_{f0} \) denotes the velocity at the exit of the perforated plate. The first order term models the enhancement of the flame velocity due to turbulence that is created by the perforated plate and is subsequently maintained inside the reaction zone by its coupling with the chemical energy release. The second-order term is a limiting term to the growth of flame velocity. It can be associated with the impact of turbulent quenching and other expansion effect. The final higher order term is used to take into account the gasdynamic effect due to the adiabatic compression by the transmitted shock. If we normalized the above equation with \( \tilde{S}_{f0} \), this yields:

\[
S_f^n = 1 + \Gamma S_f^{n-1} - \Omega S_f^{n-1^2} + \Delta S_f^{n-1^3},
\]

(6)

where \( \Gamma \) for the turbulence term, \( \Omega \) for the quenching term and \( \Delta \) for the gasdynamics term are constant parameters, which are assumed to depend physically on the initial and boundary conditions of the system. It is well-known that the logistic difference equation describes many nonlinear instability features. By computing the above model equation for \( n \rightarrow \infty \) and looking at how the system behaves, we can study the solution to different combustion regimes downstream of the perforated plate.

4. Numerical results

In general, the transmitted shock downstream of the perforated plate is weak and thus the contribution of the gasdynamic effect by the shock to the burning velocity is not significant, i.e., \( \Delta \) is relatively small and assumed to have a value of 0.08. Therefore, we only consider \( \Gamma \) and \( \Omega \) as bifurcation parameters and study how the turbulence affects the combustion processes.

Fig. 2 illustrates different behaviors of the system with various combinations of \( \Gamma \) and \( \Omega \). For large quenching parameter \( \Omega \) and small turbulent parameter \( \Gamma \), the solution to the recursion is unbounded negatively and
hence, quenching occurs. If we decrease $\Omega$ or increase $\Gamma$, a weakly unstable flame can continue to propagate. When $\Omega \sim \Gamma$, a stable solution is observed and this corresponds to the quasi-steady fast turbulent deflagration as observed experimentally. For large $\Gamma$, the solution of the system is once again unbounded and the physical outcome is the transition from deflagration to detonation (DDT).

To summarize, Fig. 3 illustrates different combustion regimes downstream of the perforated plate as a function of $\Gamma$ and $\Omega$. Different lines denote the boundary separating different possible combustion processes. At the upper limit of large $\Omega$, the negative feedback factors dominate such that the flame will quench or no ignition will occur downstream. At the other limit of small $\Omega$, the flow is highly self-turbulized and the strong turbulent mixing always leads to the transition to detonation. In between these limits, one can see that only a narrow

Fig. 3. A simple map showing different combustion regimes as a function of quenching $\Omega$ and turbulent $\Gamma$ parameters.
region exists for the stable quasi-steady turbulent deflagration. This thus explains why high-speed turbulent deflagrations are difficult to observe experimentally.

5. Concluding remarks

It is shown that a very simple mathematical model such as a logistic difference equation can recover the various experimentally observed combustion processes downstream of the interaction of a detonation with a perforated plate. It also illustrates how turbulence must be strong enough to overcome different quenching factors in order to re-initiate a detonation. An interesting result from the present computation is that a quasi-steady fast deflagration can exist in the solution when the turbulence term balanced with the others. This agrees with the recent experimental observation by Zhu et al. [3] where a quasi-steady turbulent deflagration propagating at half the Chapman–Jouguet (CJ) detonation velocity was demonstrated conclusively for the first time.

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